Kinematic Formulation of Energy-Efficient Train Speed Profiles

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Abstract—This paper proposes a kinematic model for optimising train speed profiles along homogeneous sections with a view to minimising energy consumption. By using the proposed model a very parsimonious, computationally light energy-saving optimisation problem suitable for real-time applications can be formulated. The model is tested on several appropriate case studies.

Keywords—railway; energy saving; eco drive.

I. INTRODUCTION

A key issue in rail management and operation is minimising energy consumption. Many research projects and papers aim to analyse this issue from a twofold perspective, i.e. technological improvements and optimal service/schedule design. Some ([1]-[3]) propose to optimise trajectories using optimal control theory; they consider traction and braking forces as control variables. A theoretical reference on the identification of motion regimes consistent with optimality conditions is provided in [4], and revisited in [5] in the proposition of a solution algorithm. A more thorough review is reported in [6]. Freight train optimisation has received interest as well ([7]-[8]), whilst other papers ([9]-[10]) deal specifically with suburban railways. Profile optimisation with minimum-maximum utilisation of regenerative energy is reported in [11] and [12]. The issue of energy optimisation as a function of signalling systems is reported in [13] and [14]. A model for minimising the mechanical consumed energy is proposed in [14]. Finally, energy-efficient timetable design is dealt with by [15] and, dynamically, by [16].

II. GENERAL FRAMEWORK

A. Introduction

Train departure/arrival times can be scheduled as a function of minimum travel time, resulting from optimal usage of train performance given safety and comfort constraints along a section of length \( L \) between two stations. This minimum travel time is usually defined through a trajectory encompassing an acceleration phase at maximum feasible acceleration rate \( a_{\max} \), a cruising phase at maximum allowed speed \( v_{\max} \) and a braking phase at maximum feasible deceleration rate \( |a_{\min}| \), consistent with comfort and safety. Such a trajectory – known also as all-out in the literature – yields the minimum travel time \( T_{\text{min}} \). If the actual travel time along this section is equal to \( T_{\text{min}} \) this represents the unique feasible solution. However, this is not a recommendable choice, since it does not allow recovery from delays or other unexpected events: for this aim, a reserve time is normally added to \( T_{\text{min}} \). In such cases, when the train leaves from the departure station with a delay lower than the reserve time, the kinematic trajectory of the train can be optimised in order to minimise energy consumption.

B. Notation

For greater clarification, the following notation system is used:

\[ T \] is the scheduled travel time from stations \( S_1 \) and \( S_2 \);
\[ t_{\text{acc}}, t_{\text{cr}}, t_{\text{cs}}, t_{\text{dec}} \] are the durations of the acceleration, cruising, coasting and braking phases;
\[ F(t) \] is the tractive effort at time \( t \);
\[ R(t) \] represents the resistance to motion at time \( t \);
\[ r_0, r_1, r_2 \] are the parameters of the resistance function;
\[ M \] is the mass of the train;
\( i \) is the section slope;
\( v(t) \) is the speed at time \( t \);
\( v_{\text{cr}} \) is the cruising speed;
\( v_b \) is the speed at the beginning of the braking phase equal to the speed at the end of the coasting phase;
\( s(t) \) is the space covered at time \( t \);
\( s_{\text{acc}}, s_{\text{cr}}, s_{\text{cs}}, s_{\text{dec}} \) are the spaces covered during the respective acceleration, cruising, coasting and braking phases;
\( a, a' \) are, respectively, the maximum and minimum acceleration based on technological, safety and comfort constraints;
\( v_{\max} \) is the speed limit on the section;
\( E^{\text{acc}}, E'^{\text{acc}}, E^{\text{br}} \) represent energy consumption during the acceleration, cruising and braking phases, respectively.

C. Energy optimisation problem

In order to formulate this energy optimisation problem, let \( T \) be the travel time from stations \( S_1 \) and \( S_2 \), \( F(t) \) and \( R(t) \) the motion forces and the resistances at time \( t \) respectively, \( a(t) \) and \( v(t) \) acceleration and speed at time \( t \) respectively, and \( s(t) \) the space position at time \( t \). Minimisation of the mechanical energy associated with train motion can be set as:
min \(E_{mech} = \int_{t_0}^{T} F(t)v(t) dt\)

with the kinematic motion constraints \(a(t) = F(t) - R(t)\), \(a(t) = dv(t)/dt\) and \(v(t) = ds(t)/dt\), the technological and comfort constraint \(a^* \leq a(t) \leq a\), the speed limit constraint \(v(t) \leq v_{max}\) on the section between \(S_1\) and \(S_2\) related to the characteristics of the infrastructure, and scheduling constraints \(s(0) = S_1\), \(s(T) = S_2\), \(v(0) = v(T) = 0\).

In this paper, we propose to solve the problem as an optimisation model based only on kinematic variables. This may be achieved under the following assumptions: (a) the sections are homogeneous; (b) the maximum speed and the maximum acceleration (and, in absolute value, maximum deceleration) are known and equal those defined for comfort and safety reasons (usually, these values ‘dominate’ those defined as a function of traction and braking forces). This approach, providing an optimal solution for each homogeneous section, can in principle be extended to non-homogeneous sections and/or corridors.

As pointed out by [1] and [4] – who proved that partial tractive and braking forces lead to energy optimisation only in constant speed regimes – both the objective function and the constraints can be expressed based only on kinematic variables. In homogeneous sections the mechanical energy can be calculated with reference to the four motion regimes (acceleration, cruising, coasting and braking) once the motion resistance functions have been defined. Usually, such resistance functions – different for locomotives and wagons – are expressed in N over kN of weight by means of quadratic functions of the train speed \(v\), that is \(R(v) = r_0 + r_1v + r_2v^2\). In the following subsections we study the four motion regimes, adopting this resistance function.

D. Acceleration

In this phase the tractive force is given by:

\[F(t) = M \cdot a + M \cdot R(v(t)) + M \cdot g \cdot i\]

The energy spent in the acceleration phase, \(E_{acc}\), can be calculated given that, in order to reach the cruising speed \(v_{cr}\), the train has to cover the space \(s_{acc} = \frac{v_{cr}^2}{2a}\) in a time \(t_{acc} = \frac{v_{cr}}{a}\) with an instantaneous speed \(v(t) = at\), yielding:

\[E_{acc} = E_i + \frac{M \cdot g \cdot i}{1000} \int_{t_0}^{v_{cr}} \left[ r_0 + r_1 \cdot a \cdot t + r_2 \cdot a^2 \cdot t^2 \right] dt + M \cdot g \cdot \frac{v_{cr}^2}{2a}\]

\[= M \cdot \frac{1}{2} \cdot v_{cr}^2 + M \cdot \frac{g \cdot i}{1000} \left[ \frac{1}{2} \cdot r_0 \cdot v_{cr}^2 + \frac{1}{3} \cdot r_1 \cdot v_{cr}^2 + \frac{1}{4} \cdot r_2 \cdot v_{cr}^2 \right] + M \cdot g \cdot \frac{v_{cr}^2}{2a}\]

where \(E_i\) represents the kinetic energy at the end of the acceleration phase.

E. Cruising

In this phase, the speed is constant and the tractive force equals the corresponding motion resistances. The space covered by the train will be \(s_{cr} = v_{cr} \cdot t_{cr}\). While the mechanical energy \(E'\) will be given by:

\[E'_{cr} = \frac{M \cdot g \cdot i}{1000} \int_{v_{cr}}^{v_{max}} \left[ r_0 + r_1 \cdot v + r_2 \cdot v^2 \right] dv + M \cdot g \cdot i \cdot v_{cr} \cdot t_{cr}\]

F. Coasting

In this phase, no tractive forces are applied and the train moves by inertia. Consistently, there is no energy consumption and train deceleration is due only to motion resistances. This condition is expressed by a differential equation whose solution describes how speed changes over time:

\[\frac{dv}{dt} = -\frac{M \cdot g}{1000} \left( r_0 + r_1v + r_2v^2 \right) - M \cdot g \cdot i \Rightarrow\]

\[-\int \frac{dv}{r_0 + 1000i + r_1v + r_2v^2} = \frac{g}{1000} \cdot t + C_1\]

\[-\frac{1}{r_2D} \cdot \arctan \left( \frac{v + K}{D} \right) = \frac{g}{1000} \cdot t + C_1\]

\[v(t) = D \cdot \tan \left( \frac{r_2Dg}{1000} \cdot t + C_1 \right) - K\]

where \(C_1\) is a constant depending on initial conditions and:

\[K = \frac{r_1}{2r_2} \cdot D = \sqrt{\frac{r_0 + 1000i}{r_2} - K^2}\]

Given the initial conditions:

\[v(0) = v'' = D \cdot \tan(C_1) - K \Rightarrow C_1 = \arctan \left( \frac{v'' + K}{D} \right)\]

and assuming:

\[Z = \frac{r_2Dg}{1000} \Rightarrow v(t) = D \cdot \tan(C_1 - Z \cdot t) - K\]

the coasting time is given by:

\[t_{cs} = \frac{C_1}{Z} - \frac{1}{Z} \cdot \arctan \left( \frac{v_k + K}{D} \right)\]

and the space \(s_{cs}\) covered under coasting is:

\[s_{cs} = \frac{D}{Z} \cdot \ln \left( \cos(C_1 - Z \cdot t_{cs}) \right) - K \cdot t_{cs} + C_2\]

with \(C_2\) constant depending on initial conditions. Assuming \(s=0\) at the beginning of the coasting phase, \(C_2\) is equal to:

\[C_2 = -\frac{D}{Z} \cdot \ln \left( \cos(C_1) \right) = \frac{D}{2Z} \cdot \ln \left( 1 + \left( \frac{v'' + K}{D} \right)^2 \right)\]

G. Braking

In this phase, a braking force is added to motion resistances. In general, if the electric traction system allows
kinetic energy to be recovered during braking, the recovered energy equals the work of the braking force whilst, if no recovery is possible (e.g. old electric or diesel traction systems), the mechanical energy balance is null.

From a kinematic viewpoint, the initial speed $v_i$ is that at the end of coasting (see section II.D), yielding a covered distance $s_i = v_i^2/2|a'|$ in a time $t_b = v_i / |a'|$, the deceleration being $a'$. Thus the mechanical energy recovered will be the initial kinetic energy at the beginning of the braking phase minus the difference of potential energy due to the slope:

$$E_b = -M \frac{1}{2} v_i^2 + M g \frac{v_i^2}{2|a'|} + \frac{M g}{1000} \left[ \frac{1}{2} r_0 \frac{v_i^2}{|a'|} + \frac{1}{3} \frac{r_1}{|a'|} v_i^3 + \frac{1}{4} r_2 \frac{v_i^4}{|a'|} \right]$$

III. EFFICIENCY OF THE TRACTION SYSTEM

The actual absorbed/recovered energy can be calculated straightforwardly from the (theoretical) mechanical energy, by accounting for the efficiency of the traction system and the braking energy recovery system. Clearly, if the efficiency is approximately constant, the expressions of the energetic consumptions $E_{acc}$ and $E_{cr}$ – respectively for acceleration and braking – have to be multiplied by the average efficiency coefficient, $\eta^a$, of the traction system, whilst the energy recovered, $E_r$, during the braking phase has to be multiplied by the average efficiency coefficient, $\eta^b$, of the recovery system.

Whilst this assumption normally holds for the braking energy recovery system, in real conditions a speed-varying efficiency $\eta(v)$ should be introduced for acceleration and cruising.

In the cruising phase, this yields:

$$E_{acc} = \frac{E^c}{\eta'(v_c)} = M g \frac{[r_a v_c + r_1 v_c^2 + r_2 v_c^3] t_{cr}}{1000} + \frac{M g i v_c t_{cr}}{\eta'(v_c)}$$

In the acceleration phase, especially for short sections, where the energy during the acceleration phase is a significant share of the overall consumed energy, the calculation of the actual energy is given by the following expression:

$$E_{acc} = \frac{1}{\eta'(v(t))} \left[ \int R(t) + M a + M g i \right] dt$$

In the case of $\eta(v)$ quadratic with coefficients $n$, $p$ and $q$, the above integral becomes:

$$E_{acc} = \frac{M g}{1000} \left[ \frac{r_a a t + r_1 a^2 t^2 + r_2 a^3 t^3}{n a^2 t^3 + p a t + q} \right] dt + \frac{M}{n a^2 t^4} \left[ (a + g) i a t \right] dt$$

which can be easily calculated in closed form, not reported here for the sake of brevity. The absorbed energy can also then be expressed as a function of the design parameter $v_{cr}$.

IV. THE PROPOSED OPTIMISATION MODEL

According to the above models, the overall train trajectory along a section between two stations encompasses the four motion phases described in Section II. Under the assumption that acceleration and deceleration are at the maximum (comfort) feasible rate, a set of possible motion trajectories (consistent with the scheduled arrival) can be defined by changing the maximum cruising speed and the duration of each motion phase. Thus, since each trajectory corresponds to a given energy consumption, the energy optimisation problem can be formulated as a function of the following design variables: cruising speed $v_{cr}$, cruising duration $t_{cr}$ and initial braking speed $v_b$, yielding

$$E = M \frac{1}{2} v_{cr}^2 + \frac{M g}{1000} \left[ \frac{1}{2} r_0 \frac{v_{cr}^2}{|a'|} + \frac{1}{3} \frac{r_1}{|a'|} v_{cr}^3 + \frac{1}{4} r_2 \frac{v_{cr}^4}{|a'|} \right] + \frac{M g i v_{cr} t_{cr}}{2a}$$

The constraint of compatibility with the train schedule can be expressed by imposing that the total space covered during the given travel time $T$ equals the distance $L$ between the two stations. Thus, summing all the terms reported in Section II we have the following constraints:

$$T = \frac{v_{cr}}{a} + t_{cr} + \frac{C_1}{Z} \frac{1}{Z} \left[ \arctan \frac{v_b + K}{D} \right] + \frac{v_b}{|a'|}$$

$$L = \frac{v_{cr}^2}{2a} + t_{cr} v_{cr} + \frac{D}{Z} \left[ \cos (C_1 - Z t_{cr}) - K t_{cr} + C_2 + \frac{v_b^2}{2|a'|} \right]$$

$$t_{cr} = t_{cr}(v_{cr}, v_b)$$

Therefore, having imposed the maximum acceleration and deceleration, the problem of minimisation of the mechanical energy can be set as an optimisation problem in the three variables $v_{cr}$, $t_{cr}$ and $v_b$, given $T$. Furthermore, the travel time constraint allows $t_{cr}$ to be calculated as a function of $v_{cr}$ and $v_b$. Taking into account the inequality constraints, the problem can then be formulated as follows:

$$[v_{cr}, v_b, t_{cr}] = \text{Arg}_{v_{cr}, v_b, t_{cr}} \min \left[ E_{acc}(v_{cr}) + E_{cr}[v_{cr}, t_{cr}(v_{cr}, v_b)] + E^c(v_{cr}) \right]$$

1 For the sake of simplicity, we refer here to the overall energy. Obviously, the discussion reported at the beginning of Section III applies.
\[ s^{acc}(v_c) + s^{cr}(v_c,v_h) + s^{cr}(v_c,v_h) + s_h(v_h) = L \]

\[ 0 \leq v_c \leq v_{\text{max}} \]

\[ v_{h,\text{min}} \leq v_h \leq v_c \]

\[ t_r \geq 0 \]

V. NUMERICAL EXPERIMENTS

In order to analyse the performance of the proposed procedure and to provide a sensitivity/robustness analysis with respect to relevant input variables, this section investigates how optimal trajectories (for isolated homogeneous sections) vary with respect to section length, train mass and reserve time \( t_r \).

Figure 1 plots the variations in energy savings (in % compared to energy consumed without optimisation) against the reserve time (in % compared to the minimum travel time, calculated on the basis of an all-out trajectory), for a 120-ton train and a section 8 km long. More specifically, the energy saving percentage obtained by adopting a trajectory without coasting, with a travel time increasing with the reserve time (% energy saved without coasting), is compared to the energy saving percentage obtained through a trajectory optimised with the procedure described in Section IV (% energy saved in optimisation). The corresponding speed profiles are reported in Figure 2.

Clearly, an increase in travel time (i.e. higher reserve time) produces a speed reduction and hence a reduction in motion resistances, leading per se to a reduction in the mechanical energy, as shown by the monotonicity of the curves in Figure 1. Furthermore, Figure 1 shows that the marginal energy saving tends to decrease with respect to the increase in the reserve time. Hence most of the energy saving can be potentially obtained with small travel time increases.

In general, in order to assess the potential of the proposed optimisation method, it is interesting to draw the difference between the curves in Figure 1 (i.e. by how much the proposed optimisation exceeds the non-optimised trajectory, where the energy saving is due only to speed reduction) for different values of the length of the section (see Figure 3) and of the train mass (see Figure 4).

As expected, the effect of the coasting phase and thus the optimisation of its duration tend to be negligible for both too short and too long sections, which are for underground and intercity services respectively. Indeed, the results reported in Figure 3 show that the proposed trajectory optimisation attains its maximum for lengths between 5 and 10 km, typical of suburban trains. In addition, it is worth noting that also the effectiveness of a coasting phase with respect to a trajectory without coasting with reduced speed tends to decrease when the reserve time increases, the effect of speed reduction (and hence of motion resistances) prevailing in such conditions.

A similar analysis, parameterised with respect to the train mass, is reported in Figure 4. In this case, energy saving variations exhibit less homogeneous variations, being nonetheless proportional to train mass.

\[ \text{Optimisation} \]
\[ \text{Without Coasting} \]

![Energy Saved vs Reserve Time](image1)

![Difference between non-optimised and optimised trajectories vs Reserve Time](image2)

![Speed vs Time](image3)
The proposed energy optimisation procedure allows ideal optimal trajectories to be identified for the train, whose actual trajectories in the real world will be slightly different from the optimised ones, for many reasons. Thus, it is important to investigate the extent to which the proposed optimised trajectory is robust with respect to uncertainty of some input parameters.

A first analysis is carried out by perturbing the train mass by means of random draws from a normal distribution, adopting coefficients of variation $cv$ equal to 0.1, 0.2 and 0.3. Then the optimal trajectory (i.e. calculated with the non-perturbed train mass) is used to simulate the train movement with the perturbed mass and the corresponding energy consumption. This value is compared with the energy consumption of the train with perturbed mass running the trajectory without coasting.

The values of energy saving underlying the optimised trajectory, for different values of the reserve time, are depicted in Figure 5, which reports the average of 30 mass draws. A similar analysis is reported in Figure 6 with reference to the robustness with respect to uncertainty in the values of the resistances. In general, the results obtained are very promising and satisfactory because, even if performance is obviously decreasing with respect to the coefficient of variation, energy saving is still significant even for non-negligible uncertainty levels.

The optimisation model described above was applied to define optimal speed profiles and to redefine timetables on the Cancelllo-Benevento regional railway in southern Italy. This railway, with 11 stations along 45 km, exhibits low traffic volumes and appears suitable for drawing up energy efficient timetables without generating interference and delays. The longest section is slightly more than 12 km and the shortest (between Benevento Appia and Benevento Centrale) about 1 km; all other sections span between 2 km and 9 km, thus falling within the potential most effective range of application of the proposed optimisation procedure. The railway is single track, with block sections corresponding to station-to-station sections. Based on the geometrical characteristics of the line, the maximum speed varies between 30 km/h and 80 km/h and the maximum slope is 1.6%. Services are operated with Firema E126 A1 trains, with a power of 720 kW, maximum speed of 120 km/h, a mass of 48.4 tons for the locomotive and 29 tons for two passenger coaches.

Assuming reserve times of 5%, 10% and 15% with respect to the minimum service time, the current timetable was re-scheduled considering the new travel times. Figures 7 and 8 report for each section and for both directions the energy savings produced by the proposed optimisation procedure, expressed in % with respect to non-optimised trajectories with the same travel time.

 VI. A REAL-SCALE CASE STUDY

Fig. 4. Difference between optimised energy saving vs. trajectory without coasting energy curves as a function of uncertainty on the train mass.

Fig. 5. Difference between optimised energy saving vs. trajectory without coasting energy curves as a function of uncertainty on the train mass.

Fig. 6. Difference between optimised energy saving vs. trajectory without coasting energy curves as a function of uncertainty on resistance parameters.
The proposed method allows speed profiles to be designed ensuring about 12%-14% of energy saving, considering a reserve time between 5% and 15%. The feasibility and robustness of new timetables were assessed through some microsimulation runs of the railway system.

VII. CONCLUSIONS

The paper proposed an approach for energy-based trajectory optimisation based only on kinematic variables, under rather mild assumptions, easily applicable both in real-time rescheduling and in the context of multiple sections where the assignment of a reserve time to the several sections has to be based on an optimisation model that takes into account the travel demand and delay probability function across the sections. Specifically, the proposed approach leads to optimal solutions in all contexts where the assumptions of maximum acceleration and deceleration hold (e.g. when the slope is consistent with keeping constant speed without partial braking).

The performance of the proposed procedure and a sensitivity/robustness analysis with respect to relevant input variables were evaluated through a case study showing how optimal trajectories and related energy consumption varied with respect to stretch length, train mass and reserve time $t_r$. The results were very promising and energy saving was still significant even for non-negligible uncertainty levels. The optimisation model is then applied to design optimal speed profiles and to redefine the timetables on the regional railway Cancello-Benevento in southern Italy.

The proposed method suggests promising research prospects. Indeed, it is already being applied in the context of a multi-objective energy optimisation problem encompassing also a demand-based term which takes into account the effects of increased travel times for each section of the railway line. This will allow a more effective distribution of the reserve times to be calculated across stretches in a corridor.

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